Saturation energy density

for laser stripping via a broad Stark state

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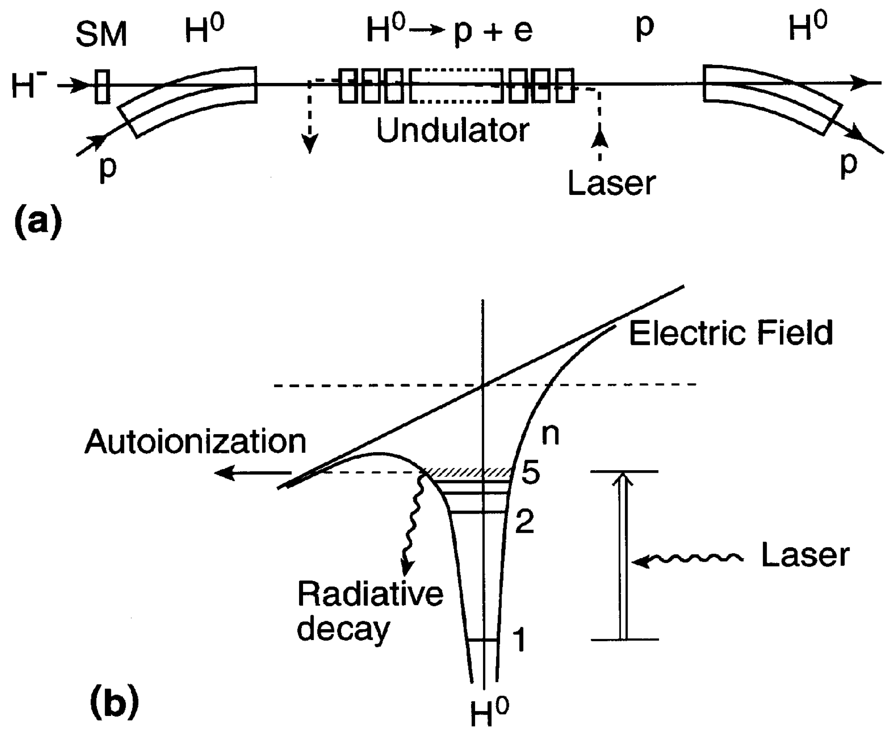
Laser Stripping via a Broad Stark State



The point at issue :
Cross section and Saturation energy density

Resonant Photoionization

Ionization Cross Section; $\sigma_i = \sigma_{kn}\eta_i$, $\eta_i \cong 1$

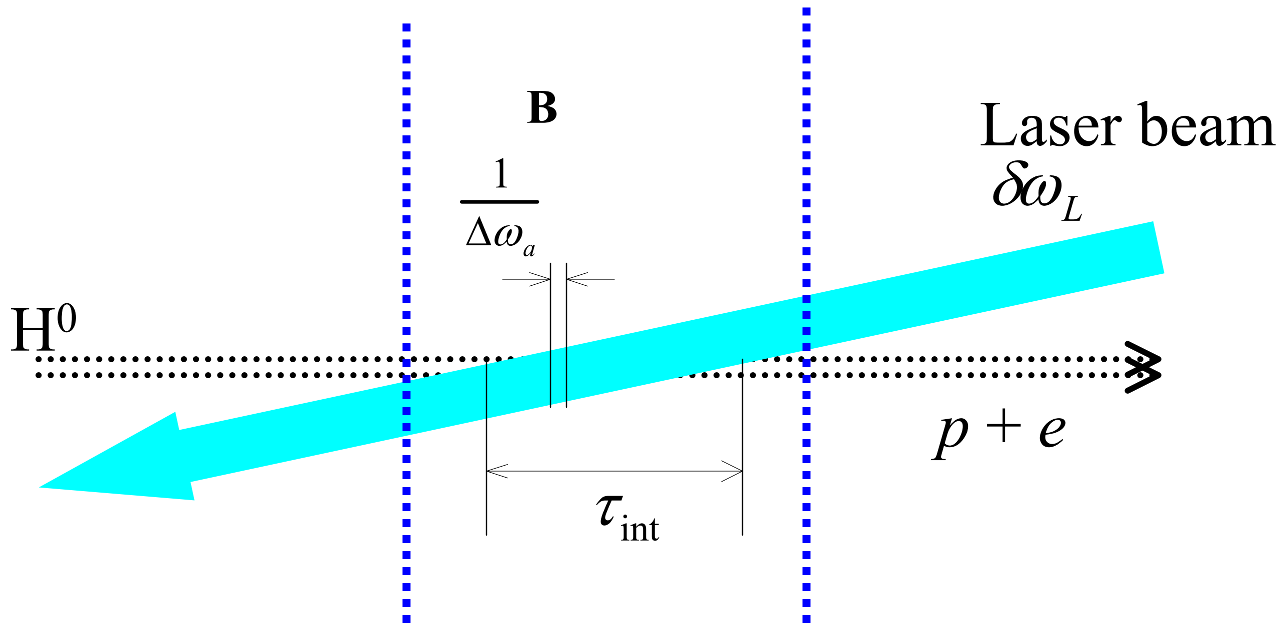
Transition Cross Section; $\sigma_{kn} = \frac{g_n}{g_k}\frac{\lambda_{kn}^2}{2\pi}\frac{A_{nk}}{\Delta\omega_a^{kn}}$,

Saturation Energy Density; $\Phi_S^{kn} = \frac{\hbar\omega_{kn}}{\sigma_{kn}}$,

Necessary Energy Density of Laser

during Interaction Period; $\Phi_{kn} = \frac{\Phi_S^{kn}}{\Delta\omega_a^{kn}\tau_{int}^{PRF}}$

Saturation Energy Density of Laser



Laser pulse for H^0 atoms in PRF

pulse width = τ_{int}/γ spectral line width ; $\Delta\omega_L = \gamma/\tau_{\text{int}}$

Cross section ; $$\sigma_{kn} = \frac{g_n}{g_k}\frac{\lambda_{kn}^2}{2\pi}\frac{A_{nk}}{\Delta\omega_a^{kn}}$$

Saturation energy density ; $$\Phi_{kn} = \left(\frac{\hbar\omega_{kn}}{\sigma_{kn}}\right)\left(\frac{\Delta\omega_L}{\Delta\omega_a^{kn}}\right) = \left(\frac{\hbar\omega_{kn}}{\sigma_{kn}}\right)\frac{\gamma}{\Delta\omega_a^{kn}\tau_{\text{int}}}$$

Comments from Dr. A. Hershcovitch

H Zero Photodetachment

On October 27, 2003, Brant Johnson and I met with Howard Bryant from the University of New Mexico to discuss Yamane's proposal and planned experiments. Prior to the meeting, I had two major issues with Yamane's proposal and planned experiments:

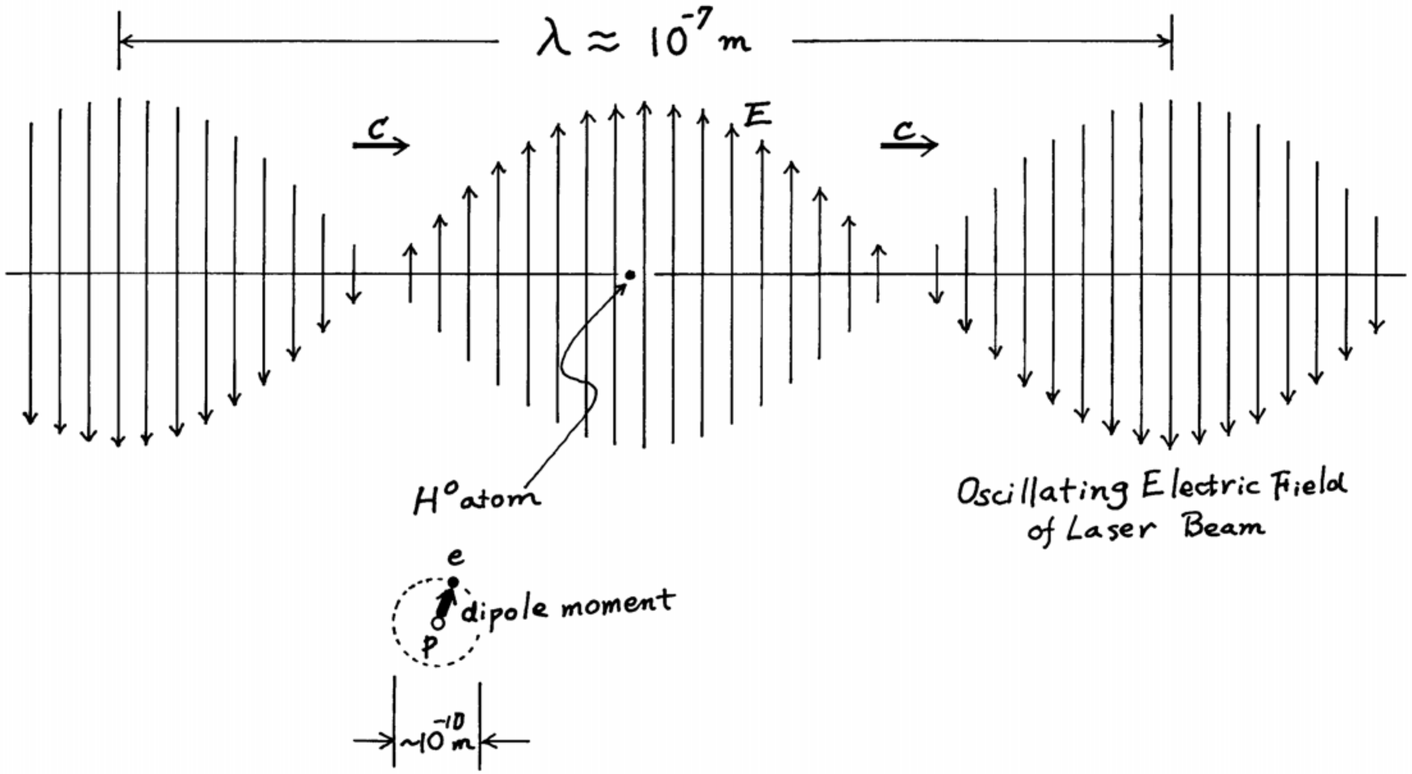
1. I had doubts about the validity equation 8 in Yamane's KEK report 2002-76 (on page 6). In equation 8, the laser energy density needed to photodetach H zero is reduced by a factor equal to the product of the line width of the transition and the interaction time in the particle rest frame. The consequence of this equation is rather critical: the laser power necessary for photodetachment is reduced by three orders of magnitude.
2. In Yamane's POP experimental proposal, photodetached protons are to be detected by a Faraday cup. If the equation 8 enhancement factor does not materialize, nothing (other than noise) would be detected by the Faraday cup.

After a rather illuminating meeting the following conclusions and/or points of discussion were reached:

1. It's a very interesting experiment; it is definitely worth performing.
2. Regarding my first concern, it is not clear that equation 8 is valid. That equation is based on equation 3.18 (on page 77) from Letokhov's book titled "Laser Photo ionization Spectroscopy." Brant and Howard expressed a major concern regarding the applicability of equation 3.18 to Yamane's case, since derivations and discussions in the book leading to this equation involve multi-electron systems. Hence it might not be applicable to the hydrogen atom.
3. Regarding my second concern, Brant and Howard suggested using a scintillator if the Faraday cup cannot detect the proton signal. Therefore, photodetached protons can be detected even if the yield is very small.
4. Concern was raised about the experiment being a true POP for SNS, since foil stripping is not equivalent to the magnetic stripping proposed for SNS. The former results in excited atoms, which later can lose the extra electron by other processes.
5. One must examine lifetime of the expected excited states in their flight path (including the magnetic fields).
6. A suggestion was made to use a laser in the first step as well.
7. A movable detector should be used after the first bending magnet, since it was found experimentally that excited hydrogen trajectories are smeared due to various stripping mechanisms. Basically, one should study trajectories of the foil-detached atoms well, before proceeding to photodetach them.

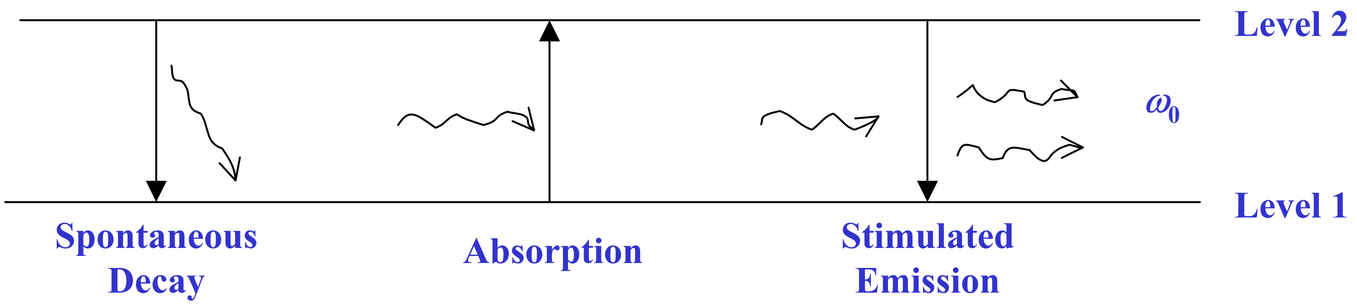
Regarding item 5, it is advisable to consult Keating et. al PRA 55, 4547 (1995). And for item 7, one must consult Gulley et. al PRA 53, 3201 (1996) especially figures 5&6; as well as Keating et al PRA 58, 4526 (1998) especially figure 5. Finally, it would be advisable to invite Brant and Howard to attend the next meeting with Yamane.

Atom Laser Interaction



Interaction between H°atom and Oscillating Electric Field of Laser

Einstein's theory on radiative transition



1, 3 types of radiation

Spontaneous Decay; $dW = A\,dt$

Absorption; $dW = B_{12}\rho(\omega)dt$

Stimulated Emission; $dW = B_{21}\rho(\omega)dt$

2, Plank's radiation law

Relation between A, B_{12}, B_{21}

$$A = \frac{\hbar\omega^3}{\pi^2 c^3} B_{21},$$

$$B_{21} = B_{12} \equiv B$$

3, Absorption, Stimulated emission

Laser power density

$$I = \int c\rho(\omega)d\omega, \qquad \rho(\omega) = \rho(\omega_0)\frac{\left(\Delta\omega_L/2\right)^2}{(\omega-\omega_0)^2 + \left(\Delta\omega_L/2\right)^2},$$

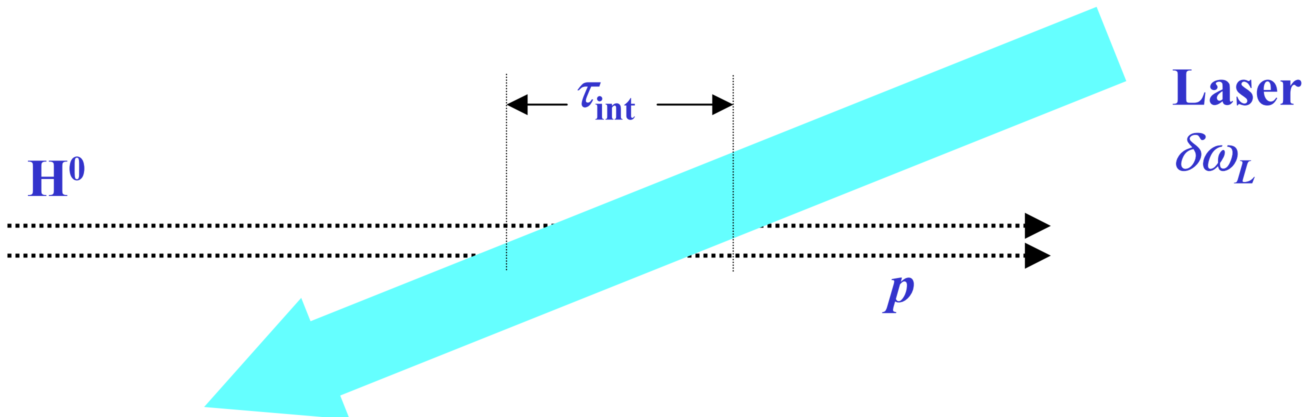
$$I = c\frac{\pi\Delta\omega_L \rho(\omega_0)}{2}$$

Saturation energy density and cross section

$$B\rho(\omega_0)\tau = B\frac{2}{c\pi\Delta\omega_L} I\tau = \frac{2c^2\pi A}{\omega_0^2\Delta\omega_L}\frac{\Phi}{\hbar\omega_0} = 1,$$

$$\Phi = \frac{\hbar\omega_0}{\left(\dfrac{\lambda^2 A}{2\pi\Delta\omega_L}\right)}, \qquad \sigma = \frac{\lambda^2 A}{2\pi\Delta\omega_L}$$

$\Delta\omega_L$: laser line width seen by H^0 atoms



$$\Delta\omega_L = \delta\omega_L + 1/\tau_{int} + \Delta\omega_a$$

$\delta\omega_L \approx 10^6 \sim 10^7$Hz ------------------- original line width of laser

$\tau_{int} \approx 10^{-9}$sec, or $1/\tau_{int} \approx 10^9$Hz --- interaction time

$\Delta\omega_a = 0$ Hz (ground state) --- level width of H^0 state
10^{13}Hz (excited state)

Velocity spread of H^0 beam does not spread $\Delta\omega_L$

Saturation energy density and cross section for absorption

Because $\Delta\omega_a = 0$, $\delta\omega_L < 1/\tau_{\text{int}}$, we have $\Delta\omega_L \approx 1/\tau_{\text{int}}$.
Thus, saturation energy density is

$$\Phi = \frac{2\pi\hbar\omega_0}{\lambda^2 A\tau_{\text{int}}}$$

This equation is same as eq.(8) in KEK Preprint 2002-76.

For the cross section, we have

$$\sigma(\omega_0) \equiv \frac{\hbar\omega_0}{\Phi} = \frac{\lambda^2 A\tau_{\text{int}}}{2\pi}$$

Saturation energy density and cross section for stimulated emission

Because H^0 is in the excited state with $\Delta\omega_a$, $\Delta\omega_L = \Delta\omega_a$.
We have

$$\Phi = \frac{2\pi\hbar\omega_0\Delta\omega_a}{\lambda^2 A}$$

This is the equation (7) of the KEK Preprint 2002-76.
For the cross section we have

$$\sigma(\omega_0) \equiv \frac{\hbar\omega_0}{\Phi} = \frac{\lambda^2}{2\pi}\frac{A}{\Delta\omega_a}$$

This is the equation (6) of the KEK Preprint 2002-76.

Derivation of the cross section formula from spontaneous decay rate: *A* of upper level

A is the rate of spontaneous decay from upper to ground state.

$$A = \int M(\omega) c\sigma(\omega) d\omega, \qquad M(\omega) = \frac{\omega^2}{\pi^2 c^3}, \qquad \sigma(\omega) = \sigma(\omega_0) \frac{\left(\Delta\omega_a/2\right)^2}{(\omega-\omega_0)^2 + \left(\Delta\omega_a/2\right)^2}.$$

$M(\omega)$ is a slowly varying function of ω.

$$A = \frac{\omega^2}{\pi^2 c^2}\sigma(\omega_0) \int \frac{\left(\Delta\omega_a/2\right)^2}{(\omega-\omega_0)^2 + \left(\Delta\omega_a/2\right)^2} d\omega = \frac{\omega^2}{\pi^2 c^2}\sigma(\omega_0)\frac{\pi\Delta\omega_a}{2} = \frac{2\pi\Delta\omega_a}{\lambda^2}\sigma(\omega_0).$$

Therefore,
$$\sigma(\omega_0) = \frac{\lambda^2}{2\pi}\frac{A}{\Delta\omega_a}.$$

This equation is same as the cross section for stimulated emission.
Thus, the cross section given by the above formula is considered to express the strength of transition that occurs during the lifetime of the upper level.

Conclusion

1. The formula (eq. (6) in KEK Preprint 2002-76)

$$\sigma\left(\omega_0\right) = \frac{\lambda^2}{2\pi}\frac{A}{\Delta\omega_a}$$

gives the cross section of transition that occurs during the lifetime of the excited state.

2. Saturation energy density for absorption in the laser stripping via a broad Stark state is given as

$$\Phi = \frac{2\pi\hbar\omega_0}{\lambda^2 A\tau_{\mathrm{int}}} = \frac{\hbar\omega_0}{\sigma\left(\omega_0\right)\Delta\omega_a\tau_{\mathrm{int}}}$$

This equation is the same as eq.(8) in KEK Preprint 2002-76